Exercise 1.46

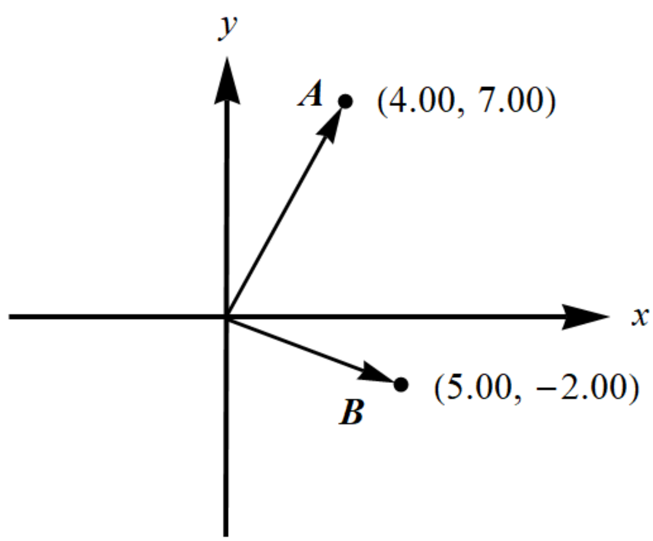
(a) Find the scalar product of the two vectors $\vec{A}$ and $\vec{B}$ given in Exercise 1.42. (b) Find the angle between these two vectors.

Solution

The two vectors from Exercise 1.42 are

$$\mathbf{A} = 4.00\hat{\boldsymbol{\imath}} + 7.00\hat{\boldsymbol{\jmath}}$$
$$\mathbf{B} = 5.00\hat{\boldsymbol{\imath}} - 2.00\hat{\boldsymbol{\jmath}}.$$

They're illustrated in the xy-plane below.



The scalar (dot) product of **A** and **B** is obtained by multiplying the respective components and adding them together.

$$\mathbf{A} \cdot \mathbf{B} = (4.00)(5.00) + (7.00)(-2.00) = 6.00.$$

The dot product can be written in terms of the angle θ between the two vectors.

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta$$

Solve for this angle.

$$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right)$$

Now that the formula for θ is known, plug in the numbers.

$$\theta = \cos^{-1}\left(\frac{6.00}{\sqrt{4.00^2 + 7.00^2}\sqrt{5.00^2 + (-2.00)^2}}\right) \approx 82.1^\circ$$